Combinatorial Abstraction Refinement for Feasibility Analysis

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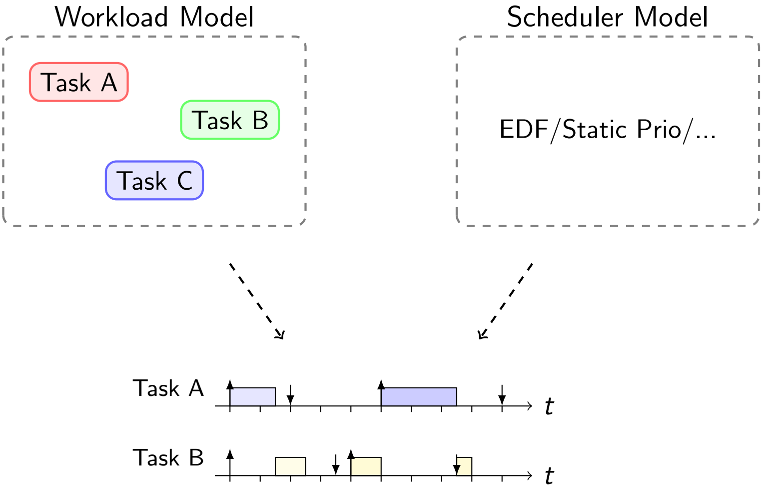
Joint work with Wang Yi

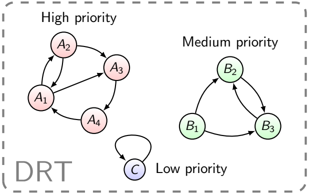
Problem Overview



Schedulable?

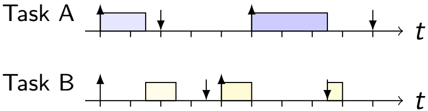
Problem Overview

Workload Model



Scheduler Model

EDF/Static Prio/...



Schedulable?

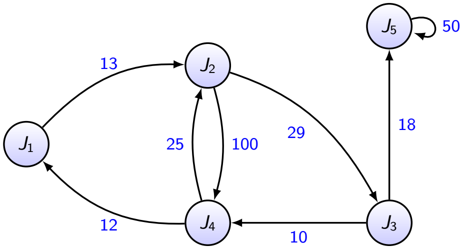
Our Setting:

- DRT tasks
- Static Priorities
- Precise Test

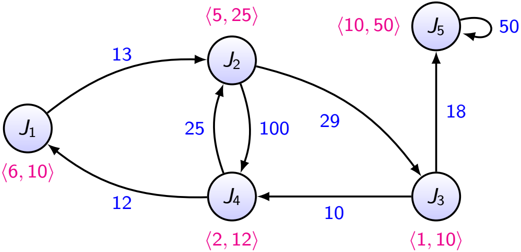
The Digraph Real-Time (DRT) Task Model

(S. et al, RTAS 2011)

- Generalizes periodic, sporadic, GMF, RRT, ...
- *Directed graph* for each task
  - Vertices J: jobs to be released (with WCET and deadline)
  - Edges (J_i, J_j): minimum inter-release delays $p(J_i, J_j)$



DRT: Semantics



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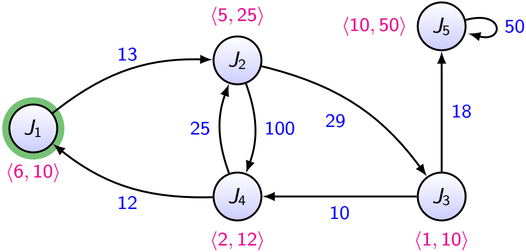

Path $\pi = (J_1)$

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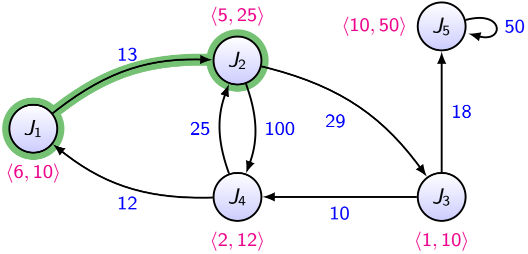
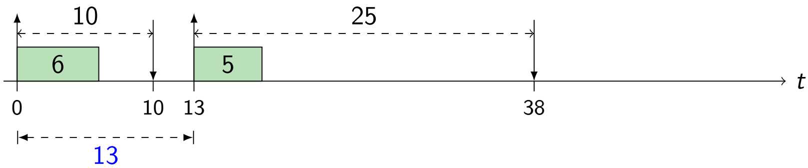


Path $\pi = (J_1, J_2)$



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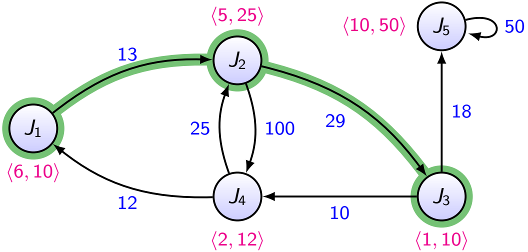
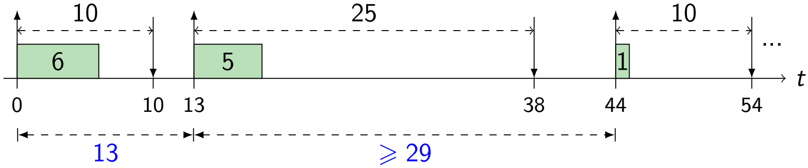


Path $\pi = (J_1, J_2, J_3)$



Complexity Results for DRT Schedulability

EDF

- *Pseudo-polynomial*
- Dbf-based analysis [RTAS 2011]
- Equivalent to Feasibility

Static Priorities

- Strongly *coNP-hard*
- Already for trees or cycles [ECRTS 2012]
- Efficient solution?

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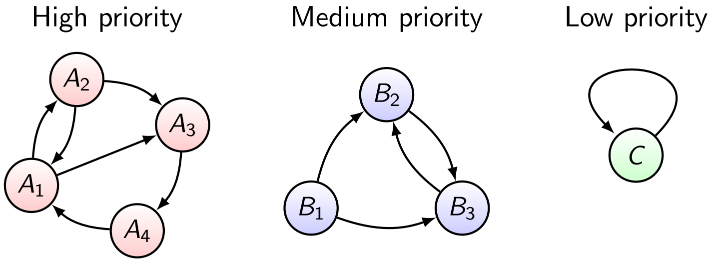
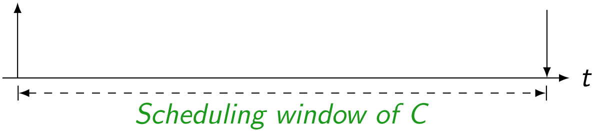
Fahrplan

Fahrplan

Testing the Scheduling Window



Is C schedulable?

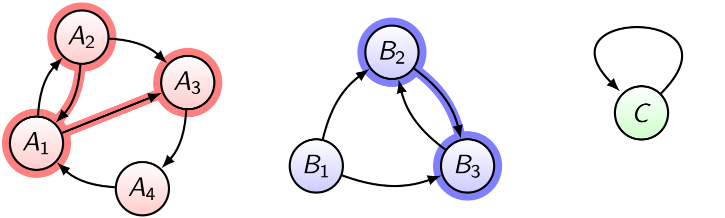


Testing the Scheduling Window

High priority

Medium priority

Low priority



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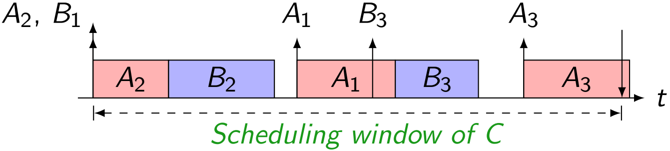
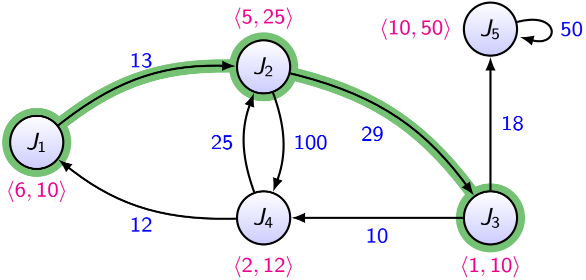
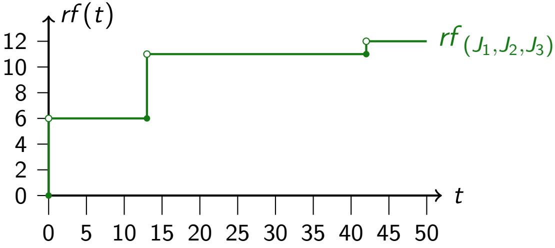


Request Functions



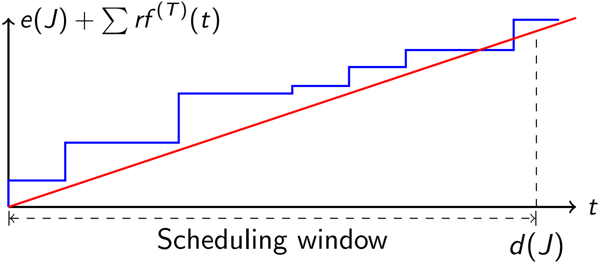


Request Functions: Schedulability Test

Lemma

A job J is schedulable iff for all combinations of request functions $rf^{(T)}$ of higher priority tasks:

$$\exists t \leqslant d(J) : e(J) + \sum_{T \in \tau} rf^{(T)}(t) \leqslant t. \quad (1)$$



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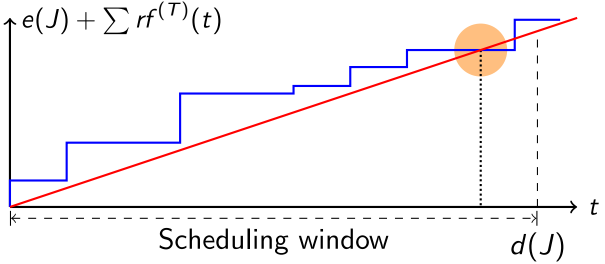

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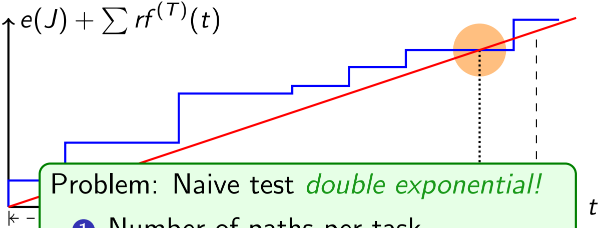
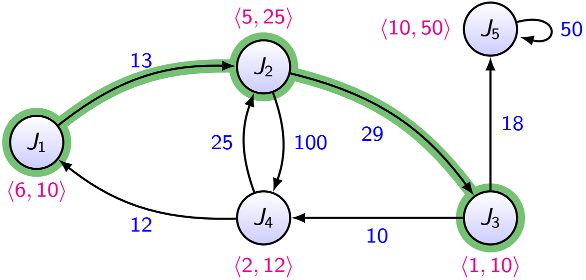
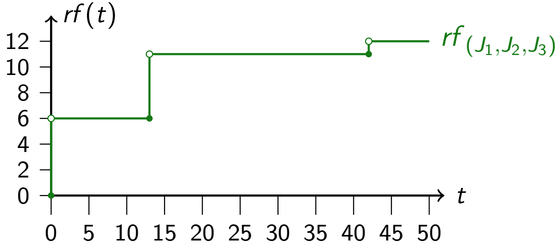


Problem: Naive test *double exponential!*

1. Number of paths per task
2. Number of combinations

Request Functions: Domination





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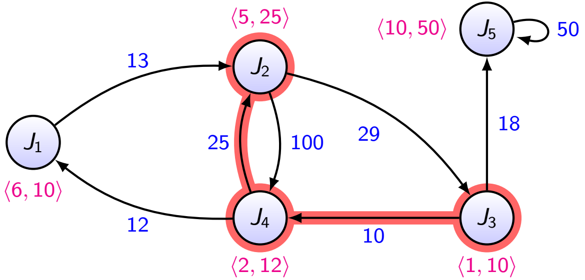

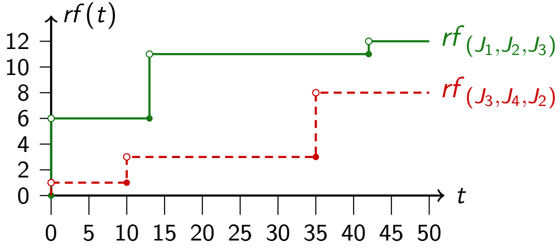

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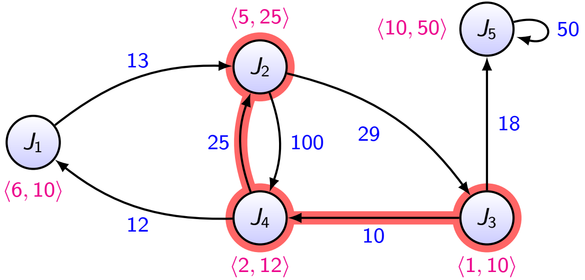

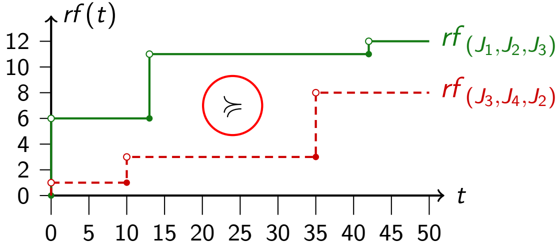

Combinatorial Explosion

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What about the *Combinatorial Explosion?*

Overapproximation: *mrf*

- Approach: Define an overapproximation
- $mrf^{(T)}(t)$: *Maximum* of *all* $rf^{(T)}(t)$ for a task T
  - "Request-Bound Function"
  - "Workload-Arrival Function"
- New test:

$$\exists t \leqslant d(J) : e(J) + \sum_{T \in \tau} mrf^{(T)}(t) \leqslant t.$$

- *Efficient*: Only *one* test, no combinatorial explosion

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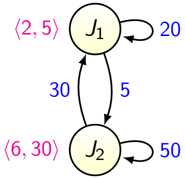
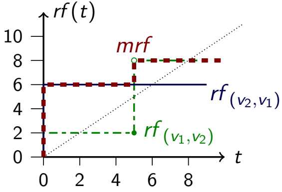
- Problem: Imprecise!





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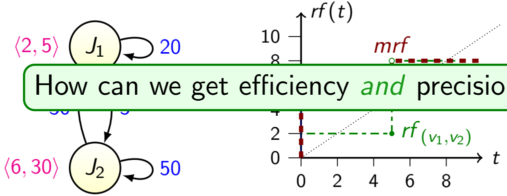
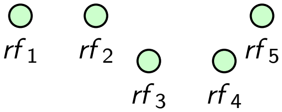


How can we get efficiency *and* precision?

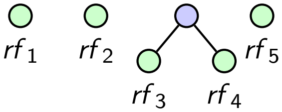
Abstraction Tree



Define an *abstraction tree* per task:

- Leaves are concrete *rf*
- Each node: maximum function of child nodes
- Root is *mrf*, maximum of *all* *rf*

Abstraction Tree



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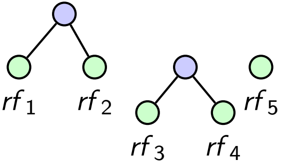

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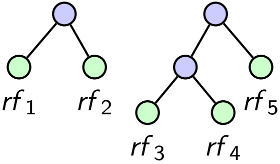

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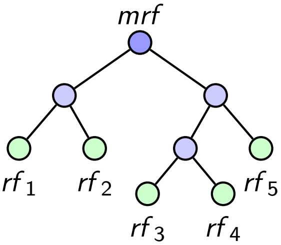

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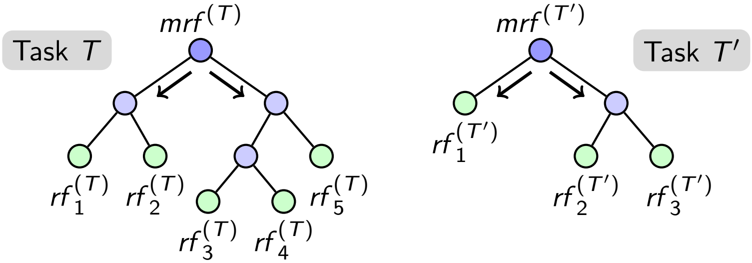
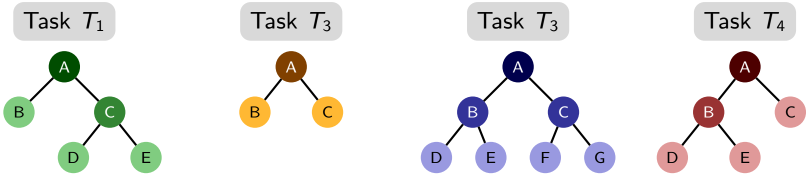
Combinatorial Abstraction Refinement



New Algorithm:

- Test *one* combination of all mrf.
- If schedulable: done
- Otherwise: Replace *one* mrf with all child nodes,
  - 2 new combinations to test
- Repeat until:
  - All combinations show schedulability, or
  - A combination of leaves shows non-schedulability

Combinatorial Abstraction Refinement: Example



Testing *rf* tuples:

(A, A, A, A)

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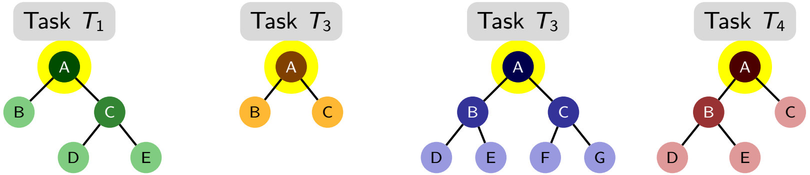

Testing *rf* tuples:

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Test: $\exists t \leqslant d(J) : e(J) + \sum_{T \in \tau} rf^{(T)}(t) \leqslant t$

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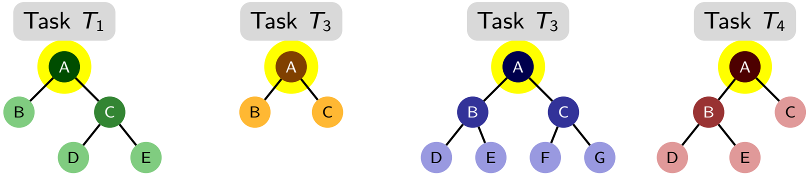
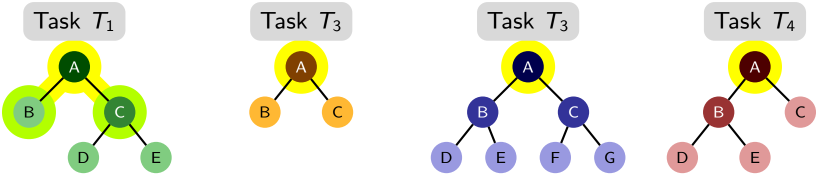


Testing *rf* tuples:

UNSCHED

Combinatorial Abstraction Refinement: Example



Testing *rf* tuples:

(A, A, A, A) **UNSCHED**

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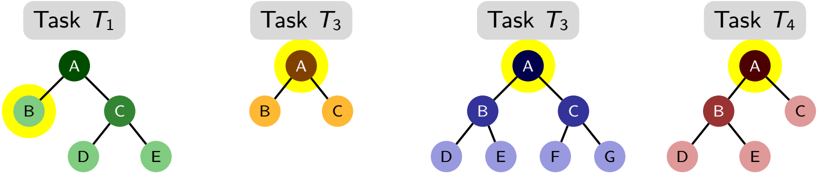
(C, A, A, A)

Combinatorial Abstraction Refinement: Example



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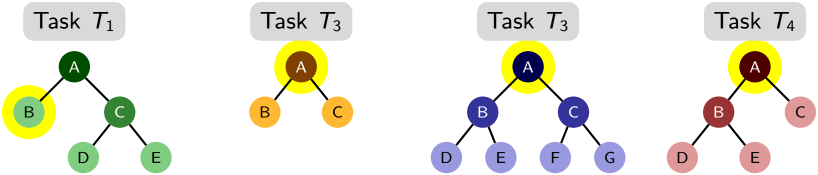

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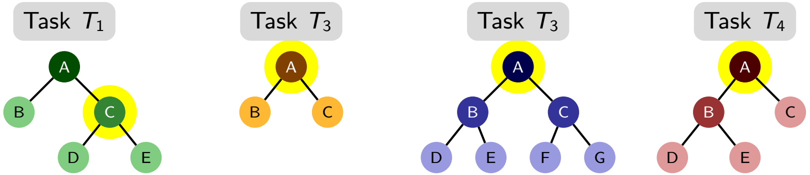
Combinatorial Abstraction Refinement: Example



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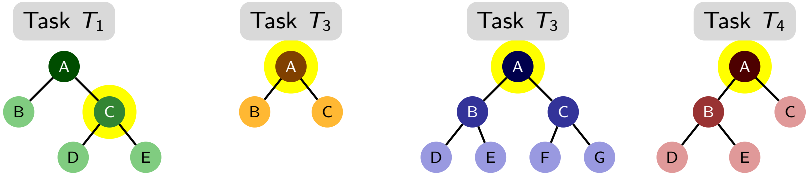

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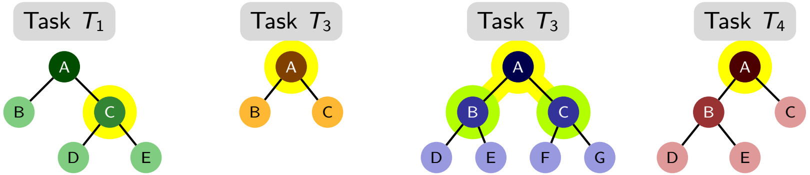

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(B, A, A, A) **SCHED**

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(C, A, B, A)

(C, A, C, A)

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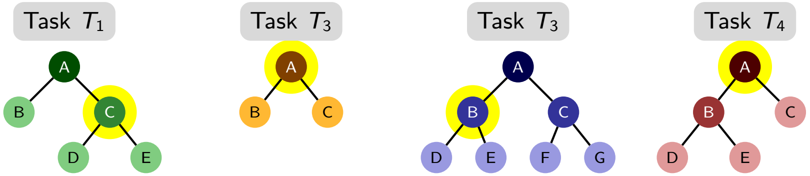

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(C, A, B, A) **?**

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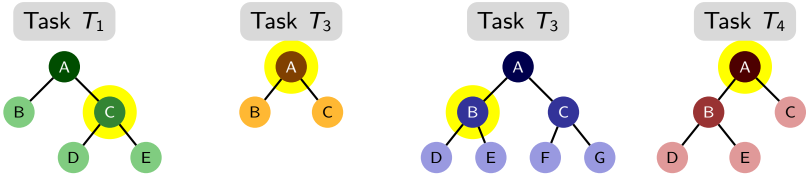

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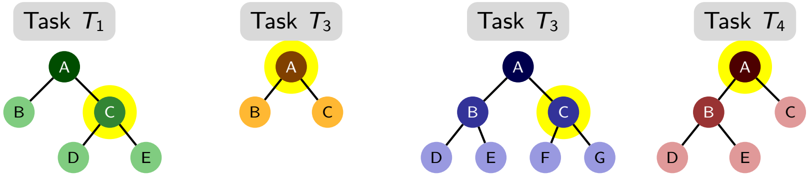

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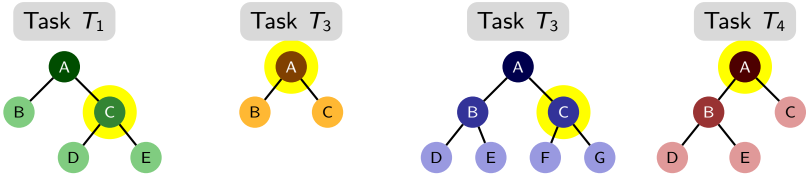

Testing *rf* tuples:

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(B, A, A, A) **SCHED**
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(C, A, B, A) **SCHED**
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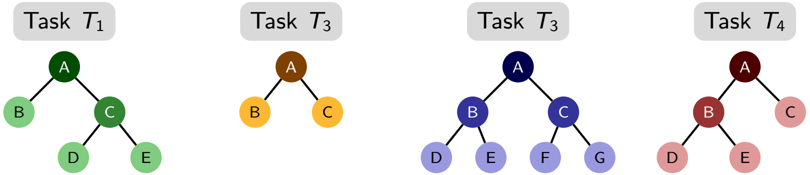
Result: *Schedulable!*

Combinatorial Abstraction Refinement: Example



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(C, A, A, A) **UNSCHED**

(C, A, B, A) **SCHED**

(C, A, C, A) **SCHED**

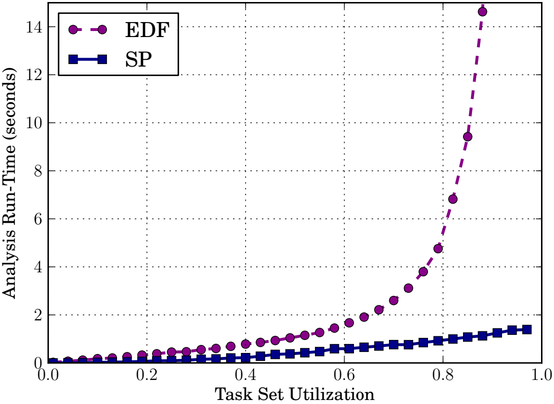
Result: *Schedulable!*

Total combinations: $3 \cdot 2 \cdot 4 \cdot 3 = 72$; Tested: 5 (!)

Fahrplan

Fahrplan

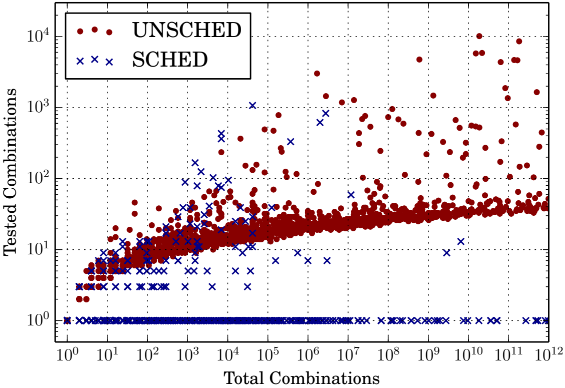
Evaluation: Runtime vs. Utilization



Comparing runtimes of

- EDF-test using dbf (pseudo-polynomial)
- SP-test based on *Combinatorial Abstraction Refinement*

Evaluation: Tested vs. Total Combinations

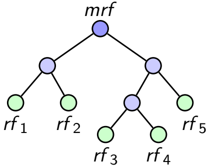


10^5 samples of single-job tests.

- Executed tests: in 99.9% of all cases, less than 100
- Total combinations possible: 10^{12} or more

Summary and Outlook

- Solve coNP-hard problem
  - Previously unsolved
  - *Efficient* method
- Abstraction refinement
  - *General* method
  - Combinatorial problems
  - Needs abstraction lattice



- Ongoing work:
  - Response-Time Analysis (submitted)
  - Apply to other problems

Q & A

Thanks!