Karlsruhe Institute of Technology

Task Set Synthesis with Cost Minimization for Sporadic Real-Time Tasks

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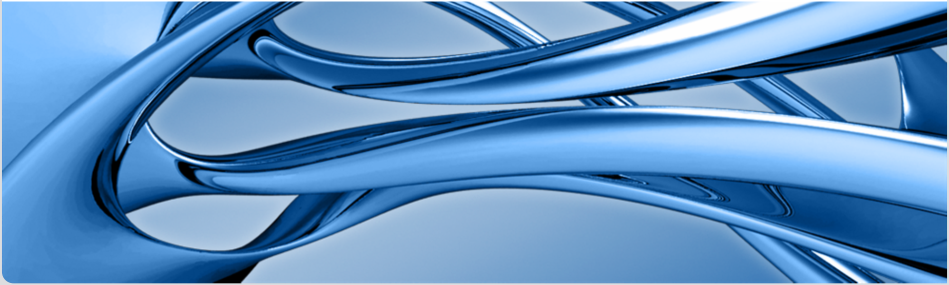

Outline

Introduction

Task Set Synthesis Problem

Proposed Combinatorial Algorithms

Conclusion

Decomposition of the Analysis

- Typical analysis and optimizations in real-time systems are decomposed into two phases
  - Phase 1: Worst-case execution time (WCET) of a stand-alone program
    - using WCET analyzers such as aiT or Chronous.
  - Phase 2: Worst-case response time of a periodic/sporadic task by considering the competition with the other tasks
    - analyzing the worst-case interference from the other tasks
    - many techniques such as utilization-based tests, response time analysis, busy-interval techniques, real-time calculus, max-plus algebra, etc.

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Sporadic Task Models

Sporadic Task τ_i:

- T_i is the minimal time between any two consecutive job releases
- A relative deadline D_i for each job from task τ_i
- (C_i, T_i, D_i) is the specification of sporadic task τ_i, where C_i is the worst-case execution time.

- implicit deadline: $D_i = T_i$, for every task τ_i,
- constrained deadline: $D_i \leq T_i$, for every task τ_i
- arbitrary deadline: otherwise

Cost-Dependent WCET

- Deriving WCET is not a simple problem
- By spending more cost, the WCET may be reduced
  - Using more SRAM in the system or larger cache size
  - Using code redundancy or execution reordering to improve the reliability
- By reducing the quality of computation, the WCET may be reduced
  - Imprecise computation
  - Multiple versions of execution plans with different qualities

QRAM Model (Rajkumar et al. RTSS'97)

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  - $\theta_i(k)$ is the cost for the k-th version of task τ_i
  - $C_i^{\theta_i(k)}$ is the corresponding WCET
  - $U_i^{\theta_i(k)} = \frac{C_i^{\theta_i(k)}}{T_i}$ as the utilization
- Without loss of generality, $\theta_i(1) < \theta_i(2) < \cdots < \theta_i(w_i)$

Output:

Select one version m_i for task τ_i such that $\mathcal{T}$ be feasibly scheduled and the system cost $\sum_{\tau_i \in \mathcal{T}} \theta_i(m_i)$ is minimized.

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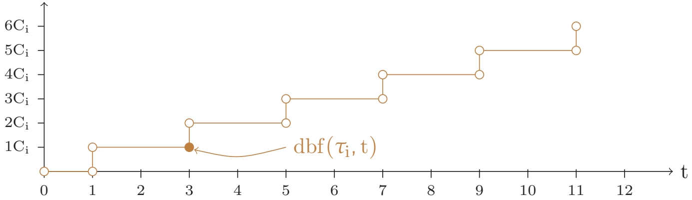
Schedulability of EDF

- Implicit deadlines: EDF is feasible if and only if the total utilization $U = \sum_{\tau_i \in \mathcal{T}} \frac{C_i}{T_i}$ is at most 100%.
- Constrained/arbitrary deadlines: demand bound testing is required



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$$\forall t \geq 0, \sum_{\tau_i \in \mathcal{T}} \mathrm{dbf}(\tau_i, t) = \sum_{i=1}^{n} \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor C_i \leq t.$$

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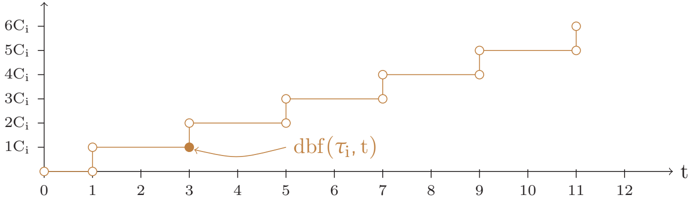

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Rate-Monotonic (RM) Scheduling

Priority Definition: A task with a smaller period has higher priority, in which ties are broken arbitrarily, i.e., $T_i \leq T_j$ if $i \leq j$.

- Least utilization upper bound for implicit deadlines: $U = \sum_{\tau_i \in \mathcal{T}} \frac{C_i}{T_i} \leq n(2^{\frac{1}{n}} - 1)$ for n tasks
- If the following condition holds, the task set is schedulable under RM:

$$\forall \tau_i \in \mathcal{T} \; \exists t \text{ with } 0 < t \leq D_i \text{ and } W_i(t) \leq t,$$

where $W_i(t)$ of task τ_i is defined as follows:

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Optimality of RM and EDF

- For uniprocessor scheduling, if there exists a feasible schedule, scheduling jobs by using EDF is also feasible.
  - EDF scheduling algorithm is optimal
- If a set of n implicit-deadline tasks, can be feasibly scheduled on a processor with a fixed-priority assignment, assigning tasks by using rate monotonic scheduling also leads to a feasible schedule.
  - RM scheduling algorithm is optimal for fixed-priority scheduling
  - Deadline Monotonic (DM) scheduling algorithm is optimal for fixed-priority scheduling

Schedulability

- The issue for uniprocessor scheduling is on how to analyze the schedulability.
- Verifying the schedulability is $\mathcal{N}$P-hard or co$\mathcal{N}$P-hard
- Approximations are possible, but what do we approximate when only binary decisions (Yes or No) have to be made?
  - Answers like probabilistic guarantee are unlikely preferred, e.g., the task set is 99% schedulable.
  - Resource augmentation by : requires a faster platform
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Resource Augmentation

For an algorithm A with a β resource augmentation factor, it guarantees that

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if the task set (system) is schedulable (feasible), Algorithm A will also returns a schedulable (feasible) answer by speeding up the system by a factor β, or

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Time Demand Function Revisit for RM

Let $w_i(t)$ of the task τ_i be defined as follows

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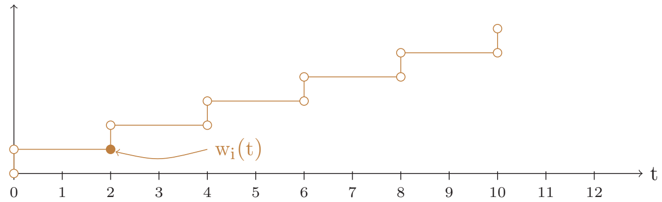
Schedulable if for each τ_i $\exists t$ with $W_i(t) \leq t$.



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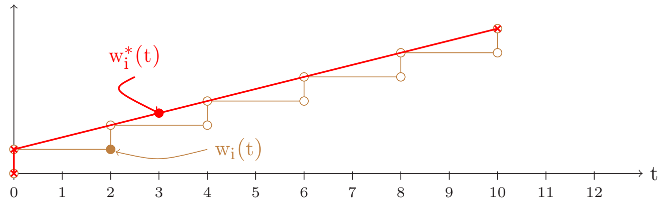

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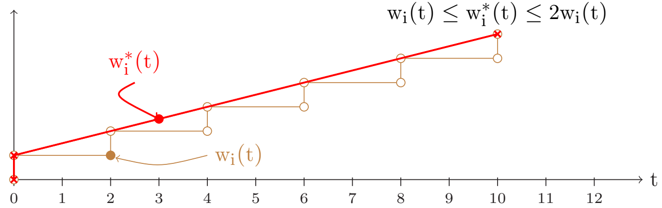

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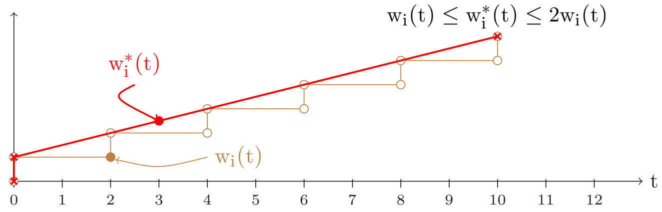
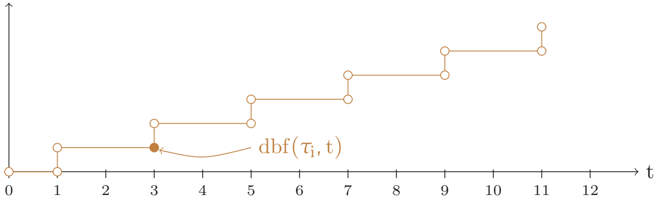


- The linear approximation makes the schedulability test easier
  - The test can be done in $O(n^2)$
  - The resource augmentation factor is 2. [Albers and Slomka ECRTS'04]

Demand Bound Function Revisit for EDF

Define demand bound function $\mathrm{dbf}(\tau_i, t)$ as

$$\mathrm{dbf}(\tau_i, t) = \max\left\{0, \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor\right\} C_i = \max\left\{0, \left\lfloor \frac{t - D_i}{T_i} \right\rfloor + 1\right\} C_i.$$



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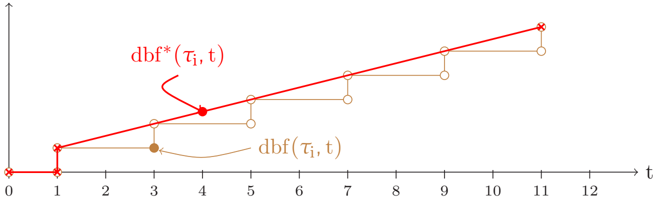

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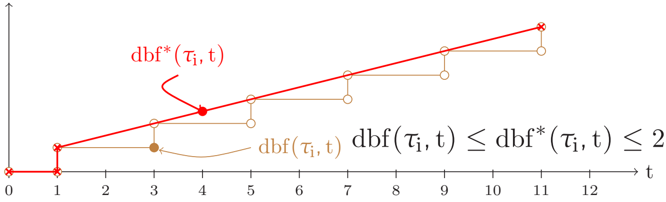

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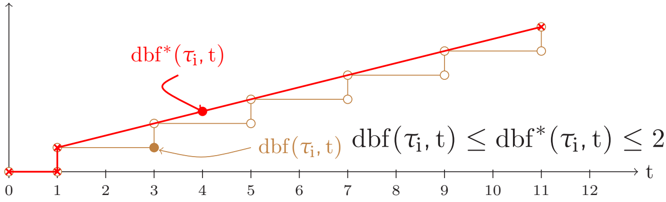

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- The linear approximation makes the schedulability test easier
  - The test can be done in $O(n^2)$
  - The resource augmentation factor is 1.6322. [Chen and Chakraborty, RTSS'11]

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Special Case for Implicit Deadlines with EDF

- The utilization bound 100% is a necessary and sufficient test.
- The problem is equivalent to the minimum multiple-choice knapsack problem
  - Given a set of items, each with w_i versions and each version has a weight and a value, the objective is to choose one version in each item such that the total weight is no more than a given limit and the total value is as small as possible.
- Many results are already known.
  - O. H. Ibarra and C. E. Kim. "Fast Approximation Algorithms for the Knapsack and Sum of Subset Problems." In: J. ACM (1975), pp. 463-468.
  - E. L. Lawler. "Fast Approximation Algorithms for Knapsack Problems". In: Math. Oper. Res. 4.4 (1979), pp. 339-356.

(α, β)-Approximation

- Suppose the optimal system cost is $B^*(I)$ for an input instance I.
- An algorithm has an α-approximation if it guarantees to have at most $\alpha \cdot B^*(I)$ system cost for any input instance I

- An (α, β)-approximation guarantees to have at most $\alpha \cdot B^*(I)$ system cost by using β speed augmentation factor.
  - With respect to speeding-up: the derived solution is a feasible solution by speeding up the platform to β, and has an α-approximation in the system cost with respect to the original instance.
  - With respect to slowing-down: the derived solution is α-approximation with respect to a problem instance by slowing down the platform to $\frac{1}{\beta}$.

An optimal algorithm for the minimum multi-choice knapsack problem:

- $(1, 1)$ for EDF with implicit deadlines
- $(1, \frac{1}{\ln 2})$ for RM with implicit deadlines

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Hardness of Approximation

Theorem

Unless $\mathcal{P} = \mathcal{NP}$, there is no polynomial-time $(\alpha, 1)$-approximation algorithm for the minimum cost synthesis problem for any $\alpha \geq 1$ when considering EDF or FP scheduling.

Proof

It is based on an L-reduction from the uniprocessor schedulability problem for sporadic real-time tasks:

- Each task has two versions
- The one with cost equals to 1 has small execution time, and another one is with "very high" cost with 50% reduction of the execution time.

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almost all the equations are different from the paper for simplicity

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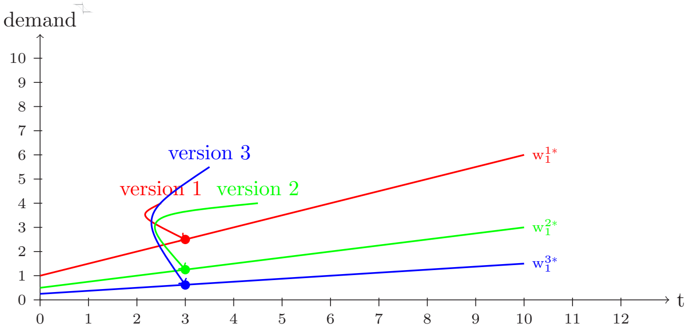

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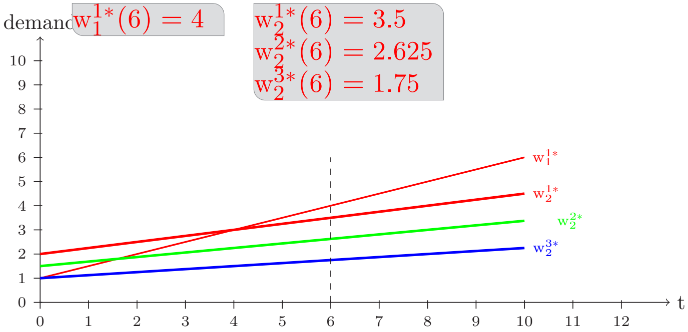

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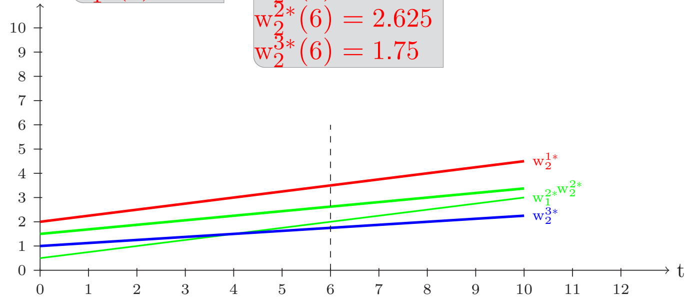

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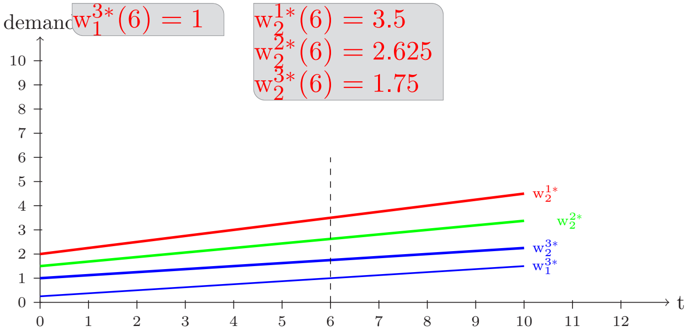

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Schedulability for DM

Deadline Monotonic (DM) is optimal when $D_i \leq T_i$.

For a given selection of versions $(m_1, m_2, \ldots, m_i)$, task τ_i can be feasibly scheduled by the DM scheduling if

$$\sum_{j=1}^{i} C_j^{\theta_j(m_j)} + D_i \cdot \sum_{j=1}^{i} U_j^{\theta_j(m_j)} \leq D_i$$

$$\Rightarrow \quad \left(\sum_{j=1}^{i-1} C_j^{\theta_j(m_j)} + D_{i-1} \cdot \sum_{j=1}^{i-1} U_j^{\theta_j(m_j)}\right) + \left(C_i^{\theta_i(m_i)} + D_i \cdot U_i^{\theta_j(m_j)} + (D_i - D_{i-1}) \sum_{j=1}^{i-1} U_j^{\theta_j(m_j)}\right) \leq D_i$$

Schedulability for DM

Deadline Monotonic (DM) is optimal when $D_i \leq T_i$.

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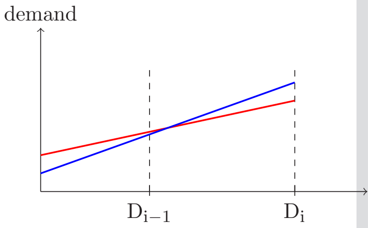
$\Rightarrow$ demand for the first $i-1$ tasks at time D_{i-1}

$$+ \left(C_i^{\theta_i(m_i)} + D_i \cdot U_i^{\theta_j(m_j)} + (D_i - D_{i-1}) \sum_{j=1}^{i-1} U_j^{\theta_j(m_j)} \right) \leq D_i$$

Dynamic Programming

- What is the minimum cost to be feasible?
- What is the minimum cost to be feasible for the first i tasks under the approximation?
  - Two terms matter: the (sub-)demand $D_i \cdot \sum_{j=1}^{i} U_j$ and (sub-)demand $\sum_{j=1}^{i} C_j^{\theta_j(m_j)}$.



Suppose that $G(i, \delta, u)$ is the minimum system cost, represented by a version selection $m_1, m_2, \ldots, m_i$, for the first i tasks such that

- the total utilization for the first i tasks is equal to u,
- the total execution time for the first i tasks is equal to $\delta \cdot D_i$, and
- $\frac{\sum_{j=1}^{k} C_j^{\theta_j(m_j)}}{D_k} + \sum_{j=1}^{k} U_j^{\theta_j(m_j)} \leq 1$ for any $k = 1, 2, \ldots, i$.

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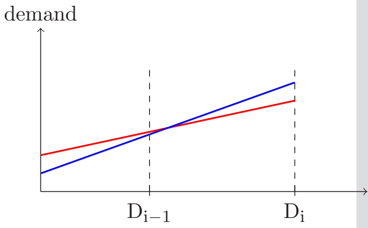

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Dynamic Programming (cont.)

- Constructing $G(i, \delta, u)$ can be done by using the standard dynamic programming approach.
  - Details [tighter definition and recursion] are in the paper
  - The minimum $G(N, \delta, u)$ for $0 \le \delta \le 1$ and $0 \le u \le 1$ has a $(1, 2)$-approximation factor for N tasks.
  - The solution is optimal on a slow-down platform with speed $\frac{1}{2}$.
- It takes pseudo-polynomial time/space for building the table

- Instead of building $G(i, \delta, u)$ for all possible values of δ and u
  - Approximate the values of interests
  - Build the table by a user-specified granularity σ
  - Lose some accuracy but earn the efficiency
  - $(1, \frac{2}{1-\eta})$-approximation with time complexity $O((\frac{N}{\eta})^2 \sum_{i=1}^{N} w_i)$ under the DM scheduling by setting σ to $\frac{1}{\lceil \frac{3N}{\eta} \rceil}$ for any given η with $0 < \eta < 1$

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EDF

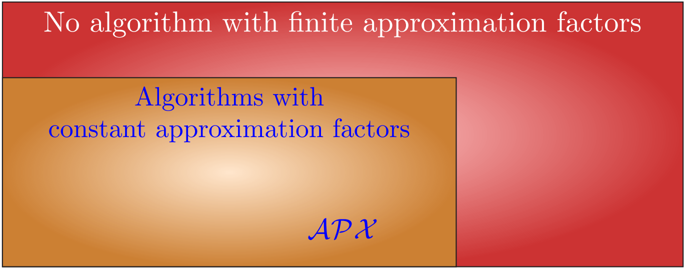
The some design philosophy also works for EDF scheduling (for arbitrary deadlines) with some minor changes.

  - Build the table by a user-specified granularity σ
  - Lose some accuracy but earn the efficiency
  - $(1, \frac{2}{1-\eta})$-approximation with time complexity $O((\frac{N}{\eta})^2 \sum_{i=1}^{N} w_i)$ under the DM scheduling by setting σ to $\frac{1}{\lceil \frac{3N}{\eta} \rceil}$ for any given η with $0 < \eta < 1$

Classification of $\mathcal{NP}$-Hard Problems

No algorithm with finite approximation factors

Classification of $\mathcal{NP}$-Hard Problems



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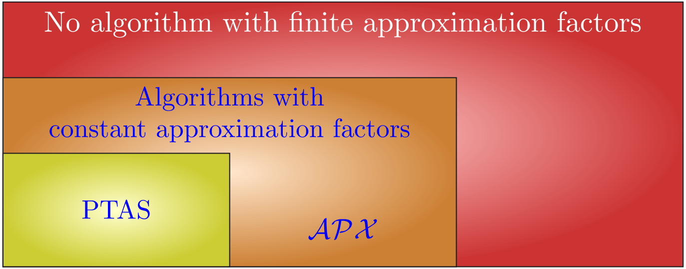

PTAS (Polynomial-time Approximation Scheme): For each constant $\epsilon > 0$, a polynomial-time partitioning algorithm A_ϵ, with approximation factor $(1 + \epsilon)$.

- complexity depends on $\frac{1}{\epsilon}$, which is assumed to be a constant, e.g., $O(n^{\frac{2}{\epsilon}})$
- allows for a trade-off of run-time versus accuracy

d-Dimensional Representative Vector Set (Chen and Chakraborty, ECRTS'12)

Among $t \in (0, \infty]$, choose $t_1, \ldots, t_d$ for density values $\frac{dbf(\tau_i, t_j)}{t_j}$ for $j = 1, \ldots, d$.

- Representative — For the accuracy
- Constant dimensions — For complexity

A d-dimensional representative vector set $\mathcal{V}$ for the given task set $\mathcal{T}$ under a user-defined tolerable error $0 < \eta$:

- for any configuration $\mathcal{T}$ and the corresponding vector set $\mathcal{V}$

$$\gamma(\mathcal{T}) \geq \max_{j=1,2,\ldots,d} \sum_{v_i \in \mathcal{V}} q_{i,j} \geq \left(\frac{1}{1+\eta}\right)\gamma(\mathcal{T}),$$

Less sampling points

Bounded error

where $\gamma(\mathcal{T})$ is the maximum density of $\mathcal{T}$.

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$(1+\epsilon, 1+\eta)$-Approximation for EDF

- Chen and Chakraborty, ECRTS'12: when $\frac{D_{max}}{D_{min}}$ is a constant, the number of representative dimensions is a constant.
- How do we achieve $(1+\epsilon, 1+\eta)$-Approximation?
  - Build a d-dimensional representative vector set $\mathcal{V}$ for $1+\eta$ speed-up guarantee
  - The problem is reduced to a minimum-cost multiple choice multiple-dimension knapsack problem
    - Set $Z = \min\{N, \lceil \frac{d}{\epsilon} \rceil\}$, bounded by a constant
    - Enumerate the combinations to select the versions for Z large tasks
    - Select the versions of the other $N - Z$ light tasks by using linear programming
    - Round the fractional variables to yield a feasible solution
    - Return the best found feasible solution as the result

Conclusion

- (α, β)-approximation for combinatorial optimization problems in RTS
  - With respect to speeding-up: the platform is speeded up to β to ensure the feasibility and optimality
  - With respect to slowing-down: the derived solution is α-approximation with respect to a problem instance by slowing down the platform to $\frac{1}{\beta}$.

	EDF	DM ($\mathrm{D_i} \leq \mathrm{T_i}$)
pseudo-polynomial	(1,1.6322)	(1,2)
polynomial	$(1, \frac{1.6322}{1-\eta})$	$(1, \frac{2}{1-\eta})$
polynomial	$(\frac{1}{1-\epsilon}, \frac{1}{1-\eta})$ $\frac{\mathrm{D_{max}}}{\mathrm{D_{min}}}$ is a constant	???????